

Brief Announcement: On the Voting Time of the Deterministic Majority Process

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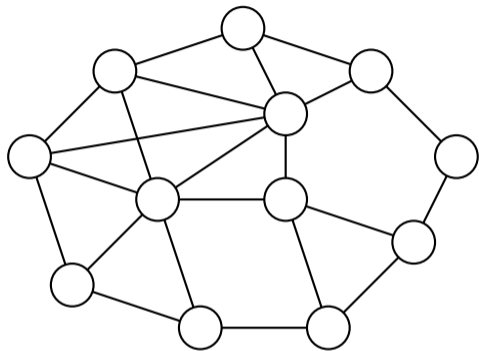
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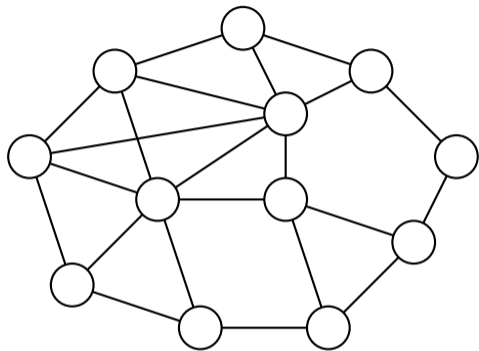
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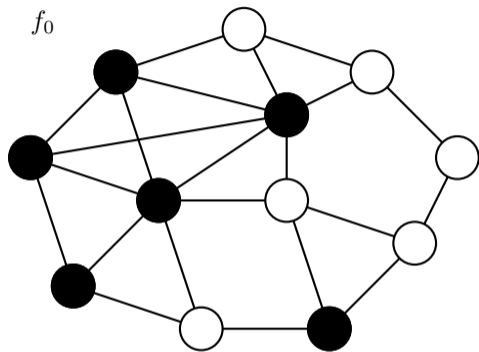
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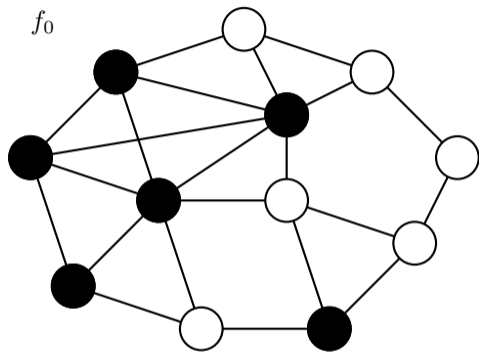
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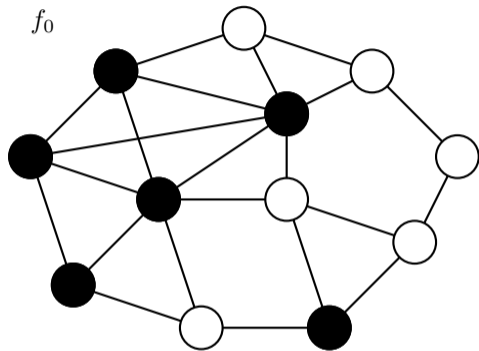
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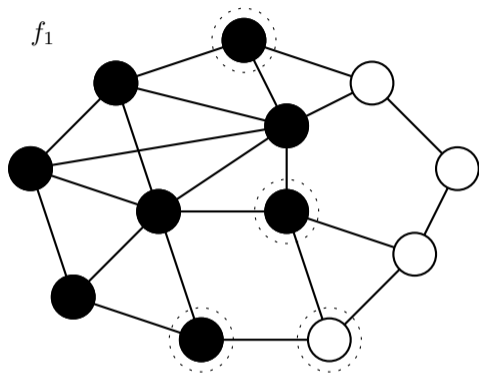
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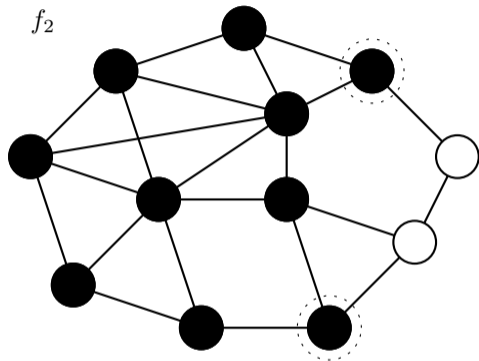
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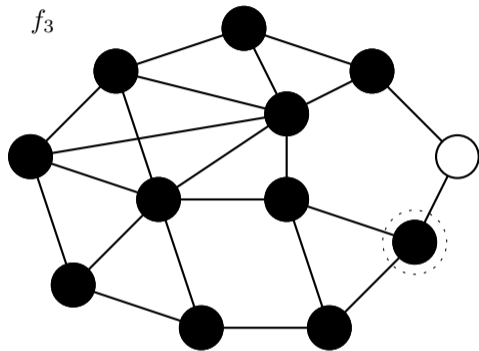
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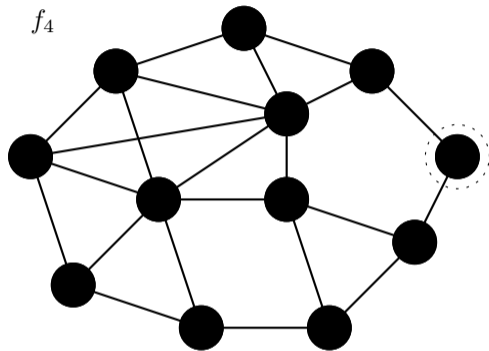
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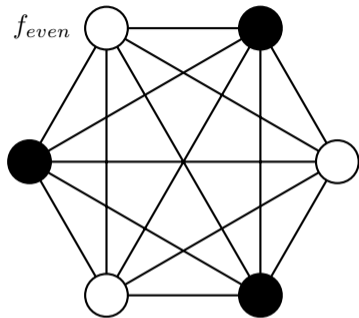
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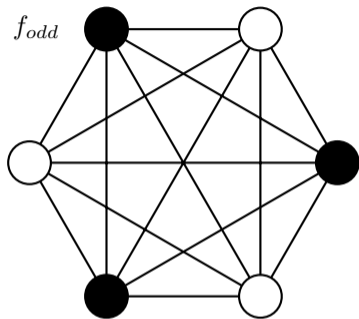
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Known Results

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- ▶ Frischknecht, Keller, Wattenhofer 2013: These bounds are tight.

Our Contribution

Bounds on the Voting Time

The voting time of the majority process is at most

$$1 + \min\{ |E| - |V_{odd}|/2, \quad |E|/2 + |V_{even}|/4 + 7/4 \cdot |V| \} .$$

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Exploiting Symmetries

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NP Hardness

For a given simple graph G and an integer k , computing whether there exists an initial opinion assignment for which the voting time of G is at least k is NP-complete.

Applications

Community Detection

Compute convergence time for distributed community detection based on label propagation clustering algorithms (Raghavan, Albert, Kumara 2007)

Thank You for Your Attention — Questions Welcome!

Introduction

Related Work

Our Contribution